	P.R.Government College (Autonomous) KAKINADA	**Program & Semester** II B.Sc. Major (IV Sem) w.e.f 2023-24 admitted batch
Course Code MAT- 403 T	**TITLE OF THE COURSE** **Integral Transforms with Applications & Problem Solving Sessions**	

Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Ordinary differential Equations and complex variables	3	1	-	3

Course Objectives:

To formalise the study of numbers and functions and to investigate important concepts such as limits and continuity. These concepts underpin calculus and its applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Understand the application of Laplace transforms to solve ODEs.
CO2	Understand the application of Laplace transforms to solve Simultaneous Des and the application of Laplace transforms to Integral equations
CO3	Basic knowledge of Fourier-Transformations
CO4	Comprehend the properties of Fourier transforms and solve problems related to finite Fourier transforms.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

Unit – I

Application of Laplace Transform to solutions of Differential Equations

Solutions of ordinary Differential Equations - Solutions of Differential Equations with constants coefficients - Solutions of Differential Equations with Variable coefficients.

Unit – II

Application of Laplace Transform to solutions of Differential Equations

Solutions of Simultaneous Ordinary Differential equations - Solutions of Partial Differential Equations.

Unit – III

Application of Laplace Transforms to Integral Equations

Definitions of Integral Equations - Abel's Integral Equation - Integral Equation of Convolution Type - Integral Differential Equations - Application of L.T. to Integral Equations.

Unit IV:

Fourier Transforms - I

Definition of Fourier Transform - Fourier sine Transform - Fourier cosine Transform - Linear Property of Fourier Transform - Change of Scale Property for Fourier Transform - sine Transform and cosine transform shifting property - Modulation theorem.

Unit – V

Fourier Transforms – II

Definition of Convolution - Convolution theorem for Fourier transform - Parseval's Identity -
Relationship between Fourier and Laplace transforms - problems related to Integral Equations -Finite Fourier Transforms - Finite Fourier Sine Transform - Finite Fourier Cosine Transform – Inversion formula for sine and cosine transforms only - statement and related problems

Activities

Seminar/ Quiz/ Assignments/ Applications of ring theory concepts to Real life Problem /Problem Solving Sessions.

Text book

B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 44th Edition, 2017

Reference books

1. Fourier Series and Integral Transformations by Dr.S. Sreenadh and others, published by S.Chand and Co, New Delhi

2. E.M. Stein and R. Shakarchi, Fourier analysis: An introduction, (Princeton University Press,2003).

3. R.S. Strichartz, A guide to Distribution theory and Fourier transforms, (World scientific, 2003)

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	1	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-IV

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Application of Laplace Transform to solutions of Differential Equations	2	1	20
II	Application of Laplace Transform to solutions of Differential Equations	2	2	30
III	Application of Laplace Transforms to Integral Equations	1	1	15
IV	Fourier Transforms - I	1	1	15
V	Fourier Transforms - II	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M

Essay questions : 3 X 10 = 30 M

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Total Marks = 50 M

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Pithapur Rajah's Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - IV Semester
Mathematics Course XI: Integral Transform with Applications
Model Paper (w.e.f. 2024-25)

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Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer any three questions selecting at least one question from each part

Part – A **3 X 10 = 30**

1. Essay question from unit - I.
2. Essay question from unit - II.
3. Essay question from unit - II.

Part – B

4. Essay question from unit - III.
5. Essay question from unit - IV.
6. Essay question from unit - V.

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

7. Short answer question from unit – I.
8. Short answer question from unit - I.
9. Short answer question from unit – II.
10. Short answer question from unit - II .
11. Short answer question from unit – III.
12. Short answer question from unit – IV.
13. Short answer question from unit - V

Short Answer Questions

UNIT - I

Application of Laplace Transform to solutions of Differential Equations

1. Using the Laplace transform of (D^2+ 4D +5) y = 5
2. Using Laplace transform solve (D^2+ 1) y = 6 cos2t if y = 3, Dy = 1 when t = 0.
3. Solve (D + 1) y = 0 if y = y_0 when t = 0.
4. Solve $\frac{dy}{dt} + y = 1$, given y = 2 when t = 0.
5. Solve (D^2+ 1) y = 0 under the condition that y = 1, $\frac{dy}{dt} = 0$ when t = 0.
6. Solve $ty'' + 2y' + ty = 0$ if y(0) = 1 and y(π) = 0.
7. Solve the equation $t\frac{d^2y}{dx^2} + (1-2t)\frac{dy}{dx} - 2y = 0, y(0) = 1, y'(0) = 2$
8. Solve $ty'' + y' + ty = 0$ gives that y(0) = 1.

UNIT – II

Application of Laplace Transform to solutions of Differential Equations

1. Using Laplace transform solve $\frac{dx}{dt} - 2x + 3y = 0, \frac{dy}{dt} + 2x - y = 0$ given x = 8, y = 3 when t = 0.
2. Solve the equation $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$ given that x = 2, y = 0 at t = 0.
3. Solve (D^2 - 3) x – 4y = 0, x + (D^2 + 1) y = 0, t > 0 when t = 0, x = y = Dy = 0, Dx = 2.
4. If y(x, t) is a function of x and t, prove that i) $L\left\{\frac{\partial y}{\partial x}\right\} = p\bar{y}(x,p) - y(x,0)$ ii) $L\left\{\frac{\partial^2 y}{\partial t^2}\right\} = p^2\bar{y}(x,p) - p\,y(x,0) - y_t(x,0)$ where $L\{y(x,t)\} = \bar{y}(x,p) and\ y_t(x,0) = (\frac{\partial y}{\partial t})_{t=0}$.
5. Solve $\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y, y(x,0) = 6e^{-3x} which\ is\ bounded\ for\ x > 0, t > 0$
6. Solve $\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = xt\ where\ y = 0 = \frac{\partial y}{\partial t}\ at\ t = 0\ and\ y(0,t) = 0$

UNIT – III

Application of Laplace Transforms to Integral Equations

1. Solve the integral equation $F(t) = e^{-t} - 2\int_0^t \cos(t-u)\,F(u)du$.
2. Solve the integral equation $\int_0^t F(u)F(t-u)du\ = 16\ sin4t.$
3. Using Laplace transform, solve $F(t) = 1 - e^{-t} + \int_0^t \text{y}(t-u)\sin u\,du.$
4. Solve the equation $F'(t) = sint + \int_0^t F(t-u)cosu\,du,$ for with the condition that F(0) = 0
5. Solve the integral equation $\int_0^t \frac{F(u)du}{\sqrt{(t-u)}} = 1 + t + t^2$.

UNIT – IV

Fourier Transforms - I

1. Find the relation between Fourier transform and Laplace transform.
2. Find the Fourier transform of $f(x) = \begin{cases} x, |x| \le a \\ x, |x| > a \end{cases}$
3. Show that the Fourier transform of $f(x) = e^{\frac{-x^2}{2}} ise^{\frac{-p^2}{2}}$
4. Find the cosine transform of the function of f(x) if $f(x) = \begin{cases} \cos x, 0 < x < a \\ 0, x > a \end{cases}$

5. Find the sine transform of the function of f(x) if $f(x)=\begin{cases}0, 0<x<a\\ x, a\le x\le b\\ 0, x>b\end{cases}$

UNIT – V
Fourier Transforms – II

1. Show that $\int_0^\infty \frac{\cos\lambda x}{\lambda^2+1}d\lambda=\frac{\pi}{2}e^{-x}$
2. Solve the integral equation $\int_0^\infty f(x)\cos\lambda x\,dx=e^{-\lambda}$
3. Find the finite Fourier sine and cosine transform of f(x) = 1.
4. Find the finite sine transform of f(x) = x where 0 < x < 4.
5. Find the cosine transform of f(x) = sin nx in (0, π).

Essay Questions
UNIT - I
Application of Laplace Transform to solutions of Differential Equations

1. Using Laplace transform method, solve $(D^2+1)\,y = \sin t\,.\,\sin 2t$, t > 0 if y = 1, Dy = 0 when t = 0.
2. Solve $(D^2 - 3D + 2)\,y = 1- e^{2t}$, y=1, Dy = 0 when t = 0.
3. Solve $(D^2 - D - 2)\,y = 20\sin(2t)$, y = -1, Dy = 2 when t = 0.
4. Solve the equation $ty''+y'+4ty=0$ if y(0) = 3, $y'(0)=0$.
5. Solve $y''+ty'-y=0\ if\ y(0)=0, y'(0)=1$.

UNIT – II
Application of Laplace Transform to solutions of Differential Equations

1. Solve $(D - 2)\,x - (D + 1)\,y = 6e^{3t}$, $(2D - 3)\,x + (D - 3)\,y = 6e^{3t}$ if x = 3, y = 0 when t = 0.
2. Solve by Laplace transform method $(D - 2)\,x - (D - 2)\,y = 1 - 2t$, $(D^2 + 1)\,x + 2Dy = 0$ if x(0) = 0, y(0) = 0 and $x'(0)=0$.
3. Solve $(D^2 + 2)\,x - Dy = 1$, $Dx + (D^2 + 2)\,y = 0$ if x = 0 = Dx = y = Dy when t = 0.
4. Solve $\frac{\partial y}{\partial t}=2\frac{\partial^2 y}{\partial x^2}$ where y(0, t) = 0 = y(5, t) and y(x, 0) = 10 sin 4πx.
5. Solve $\frac{\partial y}{\partial t}=\frac{\partial^2 y}{\partial x^2}\ where\ y\left(\frac{\pi}{2},t\right)=0, (\frac{\partial y}{\partial x})_{x=0}=0\ and\ y(x,0)=cos3x$
6. Solve $\frac{\partial y}{\partial t}=\frac{\partial^2 y}{\partial x^2}., y(x,0)=3\ sin2\pi x, y(0,t)=0=y(1,t)$

UNIT – III
Application of Laplace Transforms to Integral Equations

1. Solve the integral equation $F(t)=1+\int_0^t \sin(t-u)\,F(u)du$ and verify your solution.
2. Solve the integral equation $\int_0^t \frac{F(u)du}{(t-u)^{\frac{1}{3}}}=t(1+t)$.
3. Solve $F'(t)=t+\int_0^t F(t-u)cosu\,du\,, F(0)=4$.
4. Convert the integral equation $F(t)=t^2-3t+4-3\int_0^t(t-u)^2\,F(u)du$ into differential and associated conditions.
5. Convert $y''(t)-3y'(t)+2y(t)=4sint, y(0)=1, y'(0)=-2$ into integral equation.

UNIT – IV
Fourier Transforms - I

1. a) State and prove change of scale property.

   b) State and prove Modulation theorem.

2. Find the Fourier transform of f(x) defined by $f(x) = \begin{cases} 1, |x| < a \\ 0, |x| > a \end{cases}$ and hence evaluate $\int_{-\infty}^{\infty} \frac{\sin pa \cos pa}{p} dp$.

3. Find Fourier sine transform of $\frac{e^{-ax}}{x}$ and hence deduce that

   $$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} \sin px \, dx = tan^{-1}\frac{p}{a} - tan^{-1}\frac{p}{b}$$

4. Find Fourier cosine transform of $\frac{e^{-ax}}{x}$ and hence deduce that

   $$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} \cos px \, dx = \frac{1}{\sqrt{2\pi}} log \frac{p^2+b^2}{p^2+a^2}$$

5. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, |x| \le 1 \\ 0, x > 1 \end{cases}$ and hence evaluate $\int_0^{\infty} (\frac{x \cos x - \sin x}{x^3}) \cos \frac{x}{2} dx$

UNIT – V
Fourier Transforms – II

1. State and prove Parseval's identity for Transforms.
2. Use Parseval's identity to prove that $\int_0^{a} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2ab(a+b)}$.
3. Solve for F(x) the integral equation $\int_0^{\infty} F(x) \sin xt \, dt = \begin{cases} 1, 0 \le t < 1 \\ 2, 1 \le t < 2 \\ 0, t \ge 2 \end{cases}$
4. Find the finite Fourier sine and cosine transform of the function f(x) = 2x, 0 < x < 4.
5. Find the finite Fourier sine and cosine transform of f(x) if $f(x) = \begin{cases} 1, 0 < x < \frac{\pi}{2} \\ -1, \frac{\pi}{2} < x < \pi \end{cases}$

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